1. $F(x) = \int_0^x f(t)\,dt = \int_0^x (at^2 + bt + c)\,dt =$

a)

$$= \left[\frac{1}{3}at^3 + \frac{1}{2}bt^2 + ct\right]_0^x = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 + cx$$

(1) $x = 1$ ist Extremum: $F'(1) = f(1) = 0$

(2) $x = -\frac{1}{4}$ ist WP : $F''(-\frac{1}{4}) = f'(-\frac{1}{4}) = 0$

(3) $F(2) = \frac{2}{3}$

$f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

(1) $a + b + c = 0$

(2) $-\frac{1}{2}a + b = 0$

(3) $\frac{8}{3}a + 2b + 2c = \frac{2}{3}$ 7

(3) $- 2 \cdot$ (1) $\frac{2}{3}a = \frac{2}{3} \Rightarrow a = 1$

$-\frac{1}{2}\cdot 1 + b = 0 \Rightarrow b = \frac{1}{2}$

$1 + \frac{1}{2} + c = 0 \rightarrow c = -\frac{3}{2}$

$$\underline{F(x) = \frac{1}{3}x^3 + \frac{1}{4}x^2 - \frac{3}{2}x = \frac{1}{12}(x^3 + 3x^2 - 18x)}$$ 4 / 11

b) $A = \int_{x_1}^{x_2} f(x)\,dx$

Nullstellen von f: $x^2 + \frac{1}{2}x - \frac{3}{2} = 0$

$$x = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{24}{4}}}{2} = \frac{-\frac{1}{2} \pm \frac{5}{2}}{2}$$

$x_1 = -\frac{3}{2}$; $x_2 = 1$

$$A = \int_{-\frac{3}{2}}^{1} f(x)\,dx = \left[\frac{1}{12}(4x^3 + 3x^2 - 18x\right]_{-\frac{3}{2}}^{1} =$$

$$= \frac{1}{12}\left[(4 + 3 - 18) - \left(-\frac{27\cdot 4}{8} + \frac{27}{4} + 27\right)\right] =$$

$$= \frac{1}{12}\left[-\frac{44}{4} - \frac{81}{4}\right] = -\frac{125}{48} \; ; \quad \underline{|A| = \frac{125}{48}}$$ 6

c) $A = F(1) - F(-\frac{3}{2})$

Bestimmtes Integral ist Differenz zweier Stammfunktions-terme! 4 / 21

2. $g(x) = x^3 + 2x \; ; \; D = \mathbb{R}$

a) $G_1(x) = \int_1^x (t^3 + 2t)\,dt = \left[\frac{1}{4} t^4 + t^2 \right]_1^x =$

$= \frac{1}{4} x^4 + x^2 - \left(\frac{1}{4} + 1 \right) =$

$= \frac{1}{4} x^4 + x^2 - \frac{5}{4}$ 5

b) $G(x) = \frac{1}{4} x^4 + x^2 + C$

G_2 hat bei $x = -2$ Wert 10:

$G(-2) \overset{!}{=} 10 = \frac{1}{4} \cdot 16 + 4 + C$

$C = 10 - 8 = 2$ 4

$G_2(x) = \frac{1}{4} x^4 + x^2 + 2$

c) $G_2(x) = 0$

$u \mathrel{\hat{=}} x^2$: $\frac{1}{4} u^2 + u + 2 = 0$

$D = 1 - 2 < 0$

keine Nullstellen! $\Rightarrow$ G_2 ist keine Integralfunktion 5

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3. $f_c(x) = \frac{1}{3}(cx^2 - 2x^3)$ $D = \mathbb{R}$; $c > 0$;

a) $f_c(x) = 0 \Leftrightarrow \frac{1}{3} \cdot x^2 (c - 2x)$

Nullstellen: $x = 0$ (2-fach - Berührung!)

$x = \frac{c}{2}$

$f_c'(x) = \frac{1}{3}(2cx - 6x^2) = \frac{2}{3}(cx - 3x^2)$

Extrema: $x = 0$

$x = \frac{c}{3}$

$f_c''(x) = \frac{2}{3}(c - 6x)$

WP: $x = \frac{c}{6}$

$f''(0) = \frac{2}{3}(c) > 0 \Rightarrow$ Min

$f''\left(\frac{c}{3}\right) = \frac{2}{3}(c - 2c) < 0 \Rightarrow$ Max

$f_c'''(x) = -4 \neq 0$ WP existiert!

$\Rightarrow N_1(0;0)$; $N_2\left(\frac{c}{2};0\right)$

Max$\left(\frac{c}{3};\frac{c^3}{81}\right)$ Min$(0;0)$

WP$\left(\frac{c}{6};\frac{c^3}{162}\right)$

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b) $c = 3$: $N_1(0;0)$; $N_2\left(\frac{3}{2};0\right)$

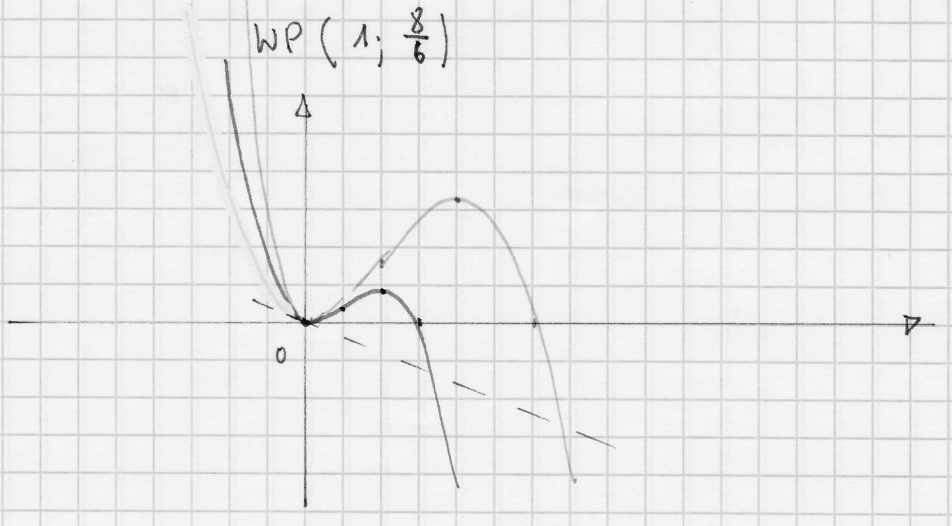
Max$\left(1;\frac{1}{3}\right)$ Min$(0;0)$

WP$\left(\frac{1}{2};\frac{1}{6}\right)$

$c = 6$: $N_1(0;0)$; $N_2(3;0)$

Max$\left(2;\frac{8}{3}\right)$ Min$(0;0)$

WP$\left(1;\frac{8}{6}\right)$



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c) Gerade g durch $P(0;0)$ und $Q(c; -\frac{1}{3}c^3)$

$$f_c(c) = \frac{1}{3}(c^3 - 2c^3) = -\frac{1}{3}c^3$$

$$g:\ y = \frac{-\frac{1}{3}c^3 - 0}{c - 0}x = -\frac{1}{3}c^2 x$$

$$y = -\frac{1}{3}c^2 x\ ; \qquad 3y = -c^2x \Leftrightarrow \underline{cx^2 + 3y = 0}\ ;$$ 4

d) $$A_c = \int_0^c (f_c(x) - g(x))\,dx = \int_0^c \left(\frac{1}{3}cx^2 - \frac{2}{3}x^3 + \frac{1}{3}c^2x\right)dx =$$

$$= \left[\frac{1}{3}c \cdot \frac{1}{3}x^3 - \frac{2}{3} \cdot \frac{1}{4}x^4 + \frac{1}{3}c^2 \cdot \frac{1}{2}x^2\right]_0^c =$$

$$= \frac{1}{9}c^4 - \frac{1}{6}c^4 + \frac{1}{6}c^4 = \frac{1}{9}c^4$$ 8

e) $$A_c \stackrel{!}{=} \frac{16}{9} \Leftrightarrow \frac{1}{\cancel{9}}c^4 = \frac{16}{\cancel{9}}$$ 4

$$c^4 = 16 \Rightarrow c = \pm 2$$

$c < 0$ nicht möglich.

Für $c = 2$ wird Flächenwert erreicht.

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