

1. $A(2|1|1)$, $B(3|0|-1)$,
 $C(4|1|0)$, $D(-2|3|6)$

a) Ebene durch A, B, C

$$\vec{x} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3-2 \\ 0-1 \\ -1-1 \end{pmatrix} + \mu \begin{pmatrix} 4-2 \\ 1-1 \\ 0-1 \end{pmatrix}$$

$$E: \vec{x} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

D ∈ E?

$$\begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

① $-2 - 2 = +\lambda + 2\mu$

② $3 - 1 = -\lambda \Rightarrow \lambda = -2$

③ $6 - 1 = -2\lambda - \mu$

① $-4 = -2 + 2\mu \Rightarrow 2\mu = -2 \Rightarrow \mu = -1$

in ③: $5 = -2(-2) - (-1) = 4 + 1 \checkmark$

$\Rightarrow D \in E$

b) Spinnrade in x_1-x_2 -Ebene: $x_3 = 0$

$$0 = 1 - 2\lambda - \mu \Rightarrow \mu = 1 - 2\lambda$$

$$\vec{x} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} + (1 - 2\lambda) \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} =$$

$$= \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix} =$$

$$\vec{x} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -1 \\ 0 \end{pmatrix} \text{ Spinnrade } x_1, x_2$$

c) $P(4|0|-2)$

$$g_{1PA}: \vec{x} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} + \sigma \begin{pmatrix} 4-2 \\ 0-1 \\ -2-1 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} + \sigma \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \quad 1$$

$$g_{2CO}: \vec{x} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + \tau \begin{pmatrix} -2-4 \\ 3-1 \\ 6-0 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + \tau \begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix} \quad 1$$

$$g_1 \neq g_2, \text{ da } \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} + \beta \cdot \begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix} \quad 2$$

$g_1 \cap g_2$

$$\begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} + \sigma \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + \tau \begin{pmatrix} -3 \\ 1 \\ 3 \end{pmatrix}$$

$$\textcircled{1} \quad 2\sigma + 3\tau = 0$$

$$\textcircled{2} \quad -\sigma - \tau = 1$$

$$\textcircled{3} \quad -3\sigma - 3\tau = 2 \quad 5$$

$$3 \cdot \textcircled{2} - \textcircled{3}: \quad 0 = -1 \quad \text{keine Schnittmenge wachschief.} \quad 9$$

d) $E_2: \vec{x} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} + \sigma \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} + \tau \begin{pmatrix} -3 \\ 1 \\ 3 \end{pmatrix}$ 3

e) KF mit Angabe

$$\vec{x} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \sigma \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \tau \begin{pmatrix} -3 \\ 1 \\ 3 \end{pmatrix}$$

$$\begin{aligned} \textcircled{1} \quad x_1 &= 2 - 2G - 3\tau && \swarrow \\ \textcircled{2} \quad x_2 &= 1 + G + \tau \Rightarrow G = x_2 - 1 - \tau && \\ \textcircled{3} \quad x_3 &= 1 - G + 3\tau && \searrow \\ &&& \quad \quad \quad 4 \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad x_1 &= 2 - 2(x_2 - 1 - \tau) - 3\tau \\ \textcircled{3} \quad x_3 &= 1 - (x_2 - 1 - \tau) + 3\tau \end{aligned}$$

$$x_1 = 2 - 2x_2 + 2 + 2\tau - 3\tau$$

$$x_3 = 1 - x_2 + 1 + \tau + 3\tau$$

$$x_1 = 4 - 2x_2 - \tau \Rightarrow \tau = 4 - 2x_2 - x_1$$

$$x_3 = 2 - x_2 + 4\tau$$

$$x_3 = 2 - x_2 + 4(4 - 2x_2 - x_1)$$

$$x_3 = 2 - x_2 + 16 - 8x_2 - 4x_1;$$

$$4x_1 + 9x_2 + x_3 = 18;$$

Schnittpunkte $x_1 \left(\frac{9}{2} \mid 0 \mid 0 \right)$

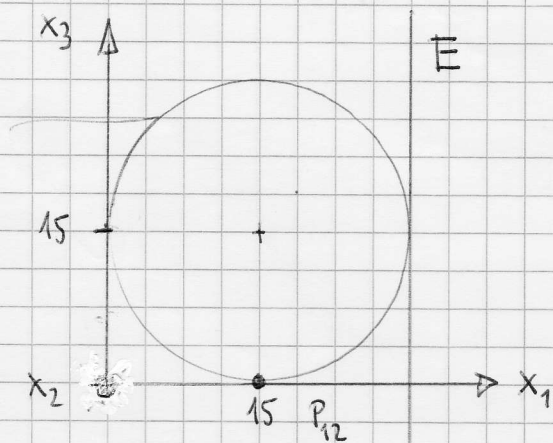
$x_2 \left(0 \mid 2 \mid 0 \right)$

$x_3 \left(0 \mid 0 \mid 18 \right)$

3

12

2. Kugel mit $r = 15$



a)

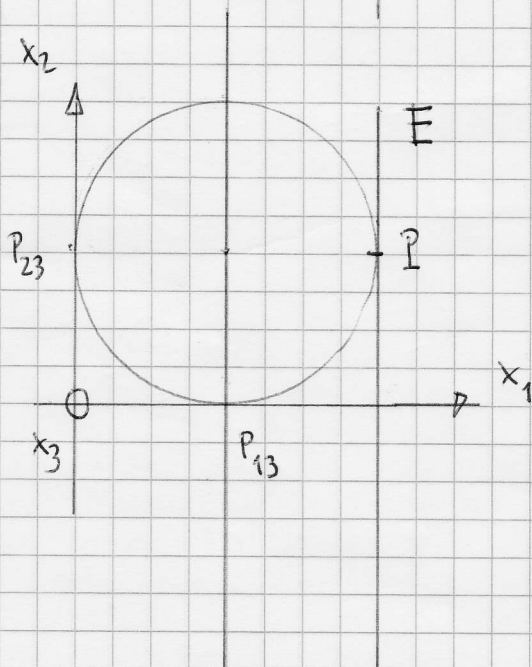
$$P_1 \begin{pmatrix} 15 & 15 & 15 \end{pmatrix} \quad 3$$

$$P_{13} \begin{pmatrix} 15 & 0 & 15 \end{pmatrix} \quad 2$$

$$P_{12} \begin{pmatrix} 15 & 15 & 0 \end{pmatrix} \quad 1$$

$$P_{23} \begin{pmatrix} 0 & 15 & 15 \end{pmatrix} \quad 1$$

/6



$$b) \quad P \begin{pmatrix} 30 & 15 & 15 \end{pmatrix} \quad 3$$

$$E: x_1 = 30 \quad 3$$

in Parameterform

$$E: \vec{x} = \begin{pmatrix} 30 \\ 15 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

/6

c) Bahn der Kugel:

$$g: \vec{x} = \begin{pmatrix} 15 \\ 15 \\ 0 \end{pmatrix} + \sigma \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

/3

15
38